Normal Distribution

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Course

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Date

Normal Distribution

1. X is a normally distributed random variable with X ~ N (30, 25).
   Find:
   a) P(X ≥ 38)

$$Z = \frac{X - \mu}{\delta}$$

Therefore, P(X ≥ 38) will be given by:

$$Z = \frac{38 - 30}{5}$$
$$= \frac{8}{5}$$
$$= (1.6)$$
$$= P(Z \geq 1.6) = 1 - P(Z \leq 1.6) = 1 - \Phi(1.6) = 1 - 0.94520$$
$$= (0.0548) \approx 0.05$$

b) $P(X \leq 23)$

$$Z = \frac{X-\mu}{\delta}$$

Therefore, $P(X \leq 23)$ will be given by:

$$Z = \frac{23-30}{5}$$
$$= -1.4$$
$$= P(Z \leq -1.4) = \Phi(-1.4) = 0.8076 \approx 0.81$$

c) $P(X \geq 21.8)$

$$Z = \frac{21.8-30}{5}$$
$$= -1.64$$
$$= P(Z \geq -1.64) = P(Z \leq 1.64) = \Phi(1.64) = 0.95$$

d) $P(24.7 \leq X \leq 40.25)$

$= P(\frac{24.7-30}{5} \leq Z \leq \frac{40.25-30}{5} = P(-1.06 \leq Z \leq 2.05)$

$= \Phi(2.05) - \Phi(-1.06) = 0.97983 - 0.14457 = 0.83525 \approx 0.84$

2. X ~ N (3.5, 0.64)

Find:

a) $P(X \leq 4)$

$Z = \frac{X-\mu}{\delta}$

$Z = \frac{4-3.5}{0.8} = 0.625$

$= P(Z \leq 0.625) = \Phi(0.625) = 0.73565 \approx 0.74$

b) $P(X < 2)$

$Z = \frac{X-\mu}{\delta}$

$Z = \frac{2-3.5}{0.8} = -1.875 = \Phi(-1.875) = 0.03005 \approx 0.03$

c) $P(2.8 \leq X \leq 5.2)$

$= P(\frac{2.8-3.5}{0.8} \leq Z \leq \frac{5.2-3.5}{0.8}) = P(-0.875 \leq Z \leq 2.125)$

$= \Phi(2.125) - \Phi(-0.875) = 0.98342 - 0.18943 = 0.79398 \approx 0.79$

3. Variance = 20, μ =?, and we have $P(X \leq 30) = 0.84$

Thus, $\delta = \sqrt{20}$. Z value that corresponds to 0.84≈ 0.99, but;

$X = Z * \delta + \mu$, implying that, $\mu = X - Z * \delta$

$30 = 0.99 * \sqrt{20} + \mu$

$\mu = 30 - 0.99 * \sqrt{20} \approx 25.57$

4. a)

Top 0.5% is equivalent to 0.005

Implying that $A_L = 1 - 0.005 = 0.995$

0.995 corresponds to a Z value of 2.57

But $X = Z * \delta + \mu = 2.57 * 13 + 56 = 8.41$

b) Lower Quantile of the marks implies 25% = 0.25

A Z score corresponding to 0.25 = -1.96. Thus;

$$X = -1.96 * 13 + 56 = 30.52$$

5. a) Mean $= \frac{\sum_{i=1}^{n} X_i}{n} = \frac{74+69+\cdots+71+66}{15} = \frac{1050}{15} = 70.00$

b) $n = 15, \bar{x} = 70, \delta = 4$

$$CI = (\bar{x} - Z_\alpha * \frac{\delta}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_\alpha * \frac{\delta}{\sqrt{n}})$$

$$CI = \bar{x} \pm Z_\alpha * \frac{\delta}{\sqrt{n}}$$

$$A_L = \frac{CI + 1}{2} = \frac{0.95 + 1}{2} = 0.975 = 1.96$$

$$70 \pm \frac{4}{\sqrt{15}} * 1.96 = (67.98, 72.02)$$